Excitation of Surface Waves on a Unidirectionally Conducting Screen*

S. R. SESHADRI†, SENIOR MEMBER, IRE

Summary—**The excitation of plane surface waves by a line source on a unidirectionally conducting 1) infinite and 2) semi-infinite screen is considered. The conditions for the existence of the surface wave and the optimum location of the line source for obtaining the highest efficiency of excitation is determined.**

Introduction

A UNIDIRECTIONALLY conducting surface is one which is perfectly conducting in a given direction and is insulating in the perpendicular direction. It is an idealization of a screen composed of closely-spaced parallel wires such that the radii of the wires and the spacing between them are small compared to wavelength. Diffraction problems involving unidirectionally conducting screens have received considerable attention in recent times, starting with the work of Toraldo di Francia[1] who studied the problem of diffraction of a plane wave by a small circular disk composed of fine parallel wires. Karp[2] has treated the problem of diffraction of a plane wave by a semi-infinite unidirectionally conducting half-plane. Hurd[3] has treated the same problem that has been treated by Karp using Fourier transform methods and has noted the existence of a surface wave field near the unidirectionally conducting half-plane. In this paper, the problem of excitation of a surface wave by a line source on a unidirectionally conducting infinite and semi-infinite screen is investigated. The optimum location of the line source for obtaining the highest efficiency of excitation is determined.

Excitation of Surface Waves on a Unidirectionally Conducting Screen

Consider a unidirectionally conducting screen located in the xy-plane where x, y, z form a right-handed rectangular coordinate system. The screen is assumed to be conducting in the ξ direction and insulating in the η direction, where

$$\xi = x \cos \alpha + y \sin \alpha$$
$$\eta = -x \sin \alpha + y \cos \alpha$$
$$z = z, \qquad 0 < \alpha < \pi/2. \tag{1}$$

An electric current line source is located at $x=0$, $z=d$ and is parallel to the y axis. It may be represented as

$$J = \hat{y}\delta(x)\delta(z-d). \tag{2}$$

In the region exterior to the screen and the source, the electric and magnetic fields satisfy the time harmonic Maxwell's equations

$$\nabla \times \boldsymbol{E} = ik\boldsymbol{H}$$
$$\nabla \times \boldsymbol{H} = -ik\boldsymbol{E}, \tag{3}$$

and on the screen the following boundary conditions are satisfied

$$E_\xi(x, y, 0) = 0 \tag{4}$$
$$H_\xi(x, y, 0^+) = H_\xi(x, y, 0^-) \tag{5}$$
$$E_\eta(x, y, 0^+) = E_\eta(x, y, 0^-). \tag{6}$$

The problem is two-dimensional and the field quantities are independent of y. Let $\boldsymbol{E}^i$, $\boldsymbol{H}^i$ denote the fields due to the line source in the absence of the screen and $\boldsymbol{E}^s$, $\boldsymbol{H}^s$ are the additional fields produced due to the presence of the screen. Hence

$$\boldsymbol{E} = \boldsymbol{E}^i + \boldsymbol{E}^s$$
$$\boldsymbol{H} = \boldsymbol{H}^i + \boldsymbol{H}^s. \tag{7}$$

Both the incident and the scattered fields are conveniently derived from the vector potential $\boldsymbol{A}$ using (3) and the relation

$$\boldsymbol{H} = \nabla \times \boldsymbol{A}. \tag{8}$$

The incident vector potential $\boldsymbol{A}^i$ due to the current source is entirely in the y direction and in view of (2), (3) and (8) it satisfies the following differential equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right] A_y{}^i(x, z) = -\delta(x)\delta(z-d). \tag{9}$$

The solution of (9) gives for the incident vector potential

$$\boldsymbol{A}^i(x, z) = \frac{\hat{y}}{2\pi}\int_{-\infty}^{\infty} \frac{i}{2\xi} e^{i\zeta x + i\xi|z-d|}\, d\zeta \tag{10}$$

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† Gordon McKay Laboratory, Harvard University, Cambridge, Mass.; on leave from Defence Electronics Research Laboratory, Hyderobad, India.

[1] G. Toraldo di Francia, "Electromagnetic cross section of a small circular disc with unidirectional conductivity," *Il Nuovo Cimento*, series X, vol. 3, pp. 1276–1284; 1956.

[2] S. N. Karp, "Diffraction of a Plane Wave by a Unidirectionally Conducting Half-Plane," Inst. Math. Sciences, New York University, N. Y., Res. Rept. No. EM108; 1957.

[3] R. A. Hurd, "Diffraction by a unidirectionally conducting half-plane," *Can. J. Phys.*, vol. 38, pp. 168–175; February, 1960.

where

$$\xi = +\sqrt{k^2 - \zeta^2} \qquad k > \zeta$$
$$\xi = +i\sqrt{\zeta^2 - k^2} \qquad k < \zeta. \tag{11}$$

ξ defined by (11) should not be confused with the coordinate denoted by the same letter in (1). On account of the unidirectional conductivity, the current on the screen and hence the vector potential $A^s(x, z)$ which gives rise to the scattered fields are in the direction ξ only.

In view of (3) and (8) it follows that

$$\left[\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial z^2} + k^2\right] A^s(x, z) = 0. \tag{12}$$

Hence, $A^s(x, z)$ may be represented as follows,

$$A^s(x, z) = \frac{1}{2\pi}\int f(\zeta) e^{i\zeta x + i\xi|z|} d\zeta \tag{13}$$

From (3), (8) and (1), it can be derived that

$$H_\xi{}^s = 0$$
$$H_\eta{}^s = \frac{\partial}{\partial z} A^s(x, z)$$
$$H_z{}^s = \sin\alpha \frac{\partial}{\partial x} A^s(x, z)$$
$$E_\xi{}^s = \frac{i}{k}\left(k^2 + \cos^2\alpha \frac{\partial^2}{\partial x^2}\right) A^s(x, z)$$
$$E_\eta{}^s = -\frac{i}{k}\cos\alpha \sin\alpha \frac{\partial^2}{\partial x^2} A^s(x, z)$$
$$E_z{}^s = \frac{i}{k}\cos\alpha \frac{\partial^2}{\partial x \partial z} A^s(x, z). \tag{14}$$

In view of (7), (13), and (14), boundary conditions (5) and (6) are automatically fulfilled. The boundary condition (4) will enable the determination of $f(\zeta)$. From (13) and (14), it is found that

$$E_\xi{}^s(x, z) = \frac{1}{2\pi}\int \frac{i\cos^2\alpha}{k}(k^2\sec^2\alpha - \zeta^2) f(\zeta) e^{i\zeta x + i\xi|z|} d\zeta. \tag{15}$$

From (10), (8) and (3), it can be shown that

$$E_\xi{}^i(x, z) = \frac{1}{2\pi}\int -\frac{k\sin\alpha}{2\xi} e^{i\zeta x - i\xi(z-d)} d\zeta \qquad z < d. \tag{16}$$

Setting $E_\xi{}^s(x, z) + E^i(x, z) = 0$ for $z = 0$ yields the expression for $f(\zeta)$ which together with (13) gives

$$A^s(x, z) = \begin{cases} \dfrac{\xi}{2\pi}\displaystyle\int_{-\infty}^{\infty} \frac{ik^2\sin\alpha}{2\xi\cos^2\alpha} \frac{1}{(\zeta^2 - k^2\sec^2\alpha)} e^{i\zeta x + i\xi(z+d)} d\zeta & z > 0 \\ \dfrac{\xi}{2\pi}\displaystyle\int_{-\infty}^{\infty} \frac{ik^2\sin\alpha}{2\xi\cos^2\alpha} \frac{1}{(\zeta^2 - k^2\sec^2\alpha)} e^{i\zeta x - i\xi(z-d)} d\zeta & z < 0. \end{cases} \tag{17}$$

From (10) and (17), all the field quantities can be obtained using (8), (3), and (1). However, since the field quantities are all independent of y, it is convenient to derive the field quantities from the y components of the electric and magnetic fields which are easily evaluated from (10), (17), (8), (3), and (1). The result is

1) $-\infty < z < 0$

$$E_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{k\xi}{2(\zeta^2 - k^2\sec^2\alpha)} e^{i\zeta x - i\xi(z-d)} d\zeta \tag{18a}$$

$$H_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{k^2\tan\alpha}{2(\zeta^2 - k^2\sec^2\alpha)} e^{i\zeta x - i\xi(z-d)} d\zeta. \tag{18b}$$

2) $0 < z < d$

$$E_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-\frac{k}{2\xi}e^{-i\xi z} - \frac{k^3\tan^2\alpha e^{i\xi z}}{2\xi(\zeta^2 - k^2\sec^2\alpha)}\right] \cdot e^{i\xi d + i\zeta x} d\zeta \tag{19a}$$

$$H_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} -\frac{k^2\tan\alpha}{2(\zeta^2 - k^2\sec^2\alpha)} \cdot e^{i\zeta x + i\xi(z+d)} d\zeta. \tag{19b}$$

3) $d < z < \infty$

$$E_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-\frac{k}{2\xi}e^{-i\xi d} - \frac{k^3\tan^2\alpha\, e^{i\xi d}}{2\xi(\zeta^2 - k^2\sec^2\alpha)}\right] \cdot e^{i\xi z + i\zeta x} d\zeta \tag{20a}$$

$$H_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} -\frac{k^2\tan\alpha}{2(\zeta^2 - k^2\sec^2\alpha)} \cdot e^{i\zeta x + i\xi(z+d)} d\zeta. \tag{20b}$$

The contour for the integrals in (18)–(20) is along the real axis in the ζ-plane indented above the singularities at $-k\sec\alpha$ and $-k$; and below the singularities at k, and $k\sec\alpha$. For $x > 0$, the integrals may be evaluated by closing the contour in the upper half of the ζ-plane as shown in Fig. 1. The contribution to the integrals is easily shown to be the sum of the residue at the pole $\zeta = k\sec\alpha$ and a branch-cut integral.

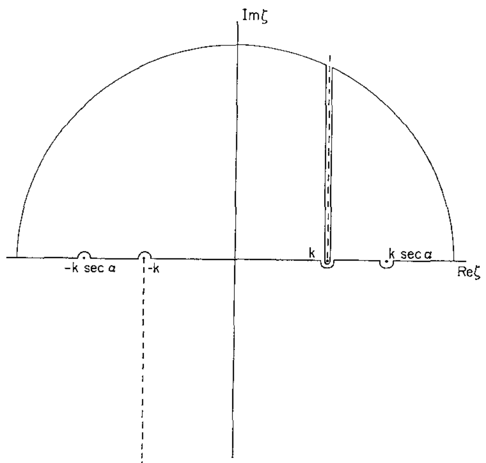

Fig. 1—Contour of integration in the ζ plane.

The value of the branch-cut integral depends on some inverse power of x and hence, for large x is negligible compared to the contribution due to the pole. Hence, for positive, large x, it results that for $z<0$

$$E_y^s(x, z) = -\frac{k \sin \alpha}{4} e^{ik \sec \alpha x + k \tan \alpha (z-d)}$$

$$H_y^s(x, z) = \frac{ik \sin \alpha}{4} e^{ik \sec \alpha r + k \tan \alpha (z-d)}$$

and for $z>0$

$$E_y^s(x, z) = -\frac{k \sin \alpha}{k} e^{ik \sec \alpha z - k \tan \alpha (z+d)}$$

$$H_y^s(x, z) = -\frac{ik \sin a}{k} e^{ik \sec \alpha z - k \tan \alpha (z+d)}. \quad (21)$$

It is evident that (21) represents a slow wave propagating in the x direction with a phase velocity which is $\cos \alpha$ times the free space velocity and is attenuated in the z direction as $e^{-k|z|\tan \alpha}$. It is easily shown with the help of (1) and (3) that the surface wave field in (21) has no ξ component of the electric and magnetic field. The concentration of the energy in the surface wave near the surface increases as α is increased. Further it is clear from (21) that the field components of the surface wave represent a right circularly polarized wave for $z<0$ and a left circularly polarized wave for $z>0$. The possibility of circularly polarized surface waves propagating along unidirectionally conducting sheets has been discussed recently by Rumsey.[4]

In order to find out the region of physical space where the surface wave in (21) is present, introduce the transformation

$$\zeta = k \cos \tau. \quad (22)$$

With (22), (18)–(20) become

1) $-\infty<z<0$

$$E_y(x, z) = \frac{1}{2\pi}\int_c -\frac{k \sin^2 \tau}{2(\cos^2 \tau - \sec^2 \alpha)} \cdot e^{ik[x \cos \tau - (z-d) \sin \tau]} d\tau \quad (23a)$$

$$H_y(x, z) = \frac{1}{2\pi}\int_c -\frac{k \tan \alpha \sin \tau}{2(\cos^2 \tau - \sec^2 \alpha)} \cdot e^{ik[x \cos \tau - (z-d) \sin \tau]} d\tau. \quad (23b)$$

2) $0<z<d$

$$E_y(x, z) = \frac{1}{2\pi}\int_c \left[e^{-ikz \sin \tau} + \frac{\tan^2 \alpha e^{ikz \sin \tau}}{(\cos^2 \tau - \sec^2 \alpha)}\right]\frac{k}{2} \cdot e^{ik[x \cos \tau + d \sin \tau]} d\tau \quad (24a)$$

$$H_y(x, z) = \frac{1}{2\pi}\int_c \frac{k \tan \alpha \sin \tau}{2(\cos^2 \tau - \sec^2 \alpha)} \cdot e^{ik[x \cos \tau + (z+d) \sin \tau]} d\tau. \quad (24b)$$

3) $\infty<z<d$

$$E_y(x, z) = \frac{1}{2\pi}\int_c \left[e^{-ikd \sin \tau} + \frac{\tan^2 \alpha e^{ikd \sin \tau}}{(\cos^2 \tau - \sec^2 \alpha)}\right]\frac{k}{2} \cdot e^{ik[x \cos \tau + z \sin \tau]} d\tau \quad (25a)$$

$$H_y(x, z) = \frac{1}{2\pi}\int_c \frac{k \tan \alpha \sin \tau}{2(\cos^2 \tau - \sec^2 \alpha)} \cdot e^{ik[x \cos \tau + (z+d) \sin \tau]} d\tau. \quad (25b)$$

The contour c in (23)–(25) is shown in Fig. 2. The asymptotic form of the field transmitted through the screen is obtained by performing a saddle-point evaluation of the integrals [(23a) and (23b)]. The saddle point which lies in the interval $0<\tau_0<\pi$ is given by

$$\tau_0 = \tan^{-1} - \frac{(z-d)}{x} \cdot \quad (26)$$

The appropriate contour through the saddle point is obtained by setting the imaginary part of the phase $ik[x \cos \tau - (z-d) \sin \tau]$ equal to its value at the saddle point $\tau = \tau_0$. For the saddle contours, it is found that

$$\tau_1 = \tau_0 \pm \cos^{-1} (\sec h\tau_2). \quad (27)$$

By requiring that the exponential in the integrand of (23a) and (23b) vanish at infinity on the contour, it results that

$$\tau_1 = \tau_0 + \cos^{-1} (\sec h\tau_2) \qquad \text{for } \tau_2 < 0$$
$$\tau_1 = \tau_0 - \cos^{-1} (\sec h\tau_2) \qquad \text{for } \tau_2 > 0. \quad (28)$$

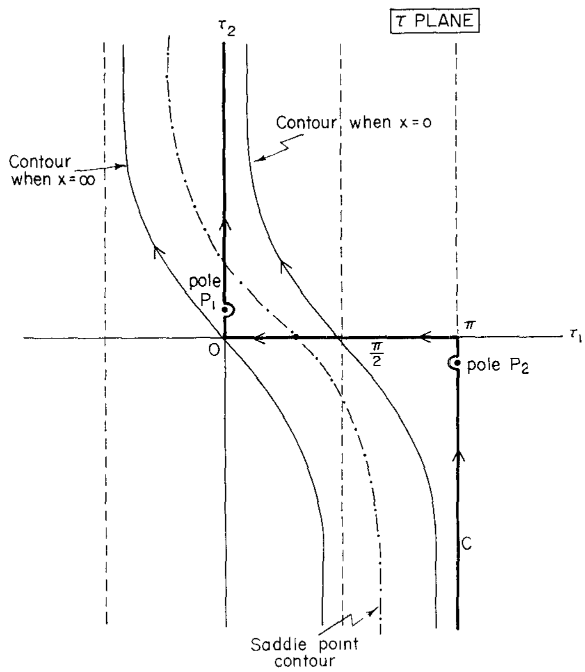

Fig. 2—Integration contours in the τ-plane $x>0$.

[4] V. H. Rumsey, "A new way of solving Maxwell's equations," IRE Trans. on Antennas and Propagation, vol. AP-9, pp. 461–465; September, 1961.

The contour c is deformed into the saddle contour (28), and the saddle-point evaluation of the integral in (23a) and (23b) gives the asymptotic form of the transmitted field. The poles of the integrand in (23a) and (23b) occur at P_1: $\tau_1=0$, $\tau_2=\cos h^{-1}\sec\alpha$ and P_2: $\tau_1=\pi$, $\tau_2=-\cos h^{-1}\sec\alpha$. For $\tau_0=\pi/2$, the original contour c can be deformed into the saddle-point contour without crossing the poles. But when $\tau_0=0$, the original contour c crosses the pole P_1 and the residue of the integral at this pole must be added to the saddle-point contribution. In order to find the condition when the surface wave is present, consider the saddle contour $\tau_1(P_1)$ which passes through the pole. From (28) it follows that

$$\tau_0(P_1) = \alpha. \tag{29}$$

If $\tau_0<\tau_0(P_1)$, the pole is crossed and the surface wave occurs; and if $\tau_0>\tau_0(P_1)$ there is no surface wave. Hence, from (26) and (29), the surface wave is seen to occur if

$$x > (d - z)\cot\alpha. \tag{30}$$

It is immediately seen from (30) that there is no surface wave for $\alpha=0$, and it occurs for very large values of x, when α is near zero. For $\alpha=\pi/2$, the screen behaves like a perfectly-conducting screen for a line current source in the y direction. But for α close to $\pi/2$, the surface waves are excited at a short distance from the source and also the energy of the surface waves is highly concentrated near the surface of the screen. For $x<0$, the contour in Fig. 1 has to be closed in the lower of the ζ-plane and the pole $\zeta=-k\sec\alpha$ will give rise to a surface wave traveling in the negative x direction. The condition for the existence of the surface wave can be found in an analogous manner. For $z>0$, either (24) or (25) (they will give identical results due to symmetry in z and d), will lead to similar results. It is verified that the surface wave is symmetrical about the planes $z=0$ and $x=0$, and (30) in the general form may be stated as follows:

$$|x| > (d + |z|)\cot\alpha. \tag{31}$$

Efficiency of Excitation

It is desirable to find the optimum height of the line source for which the part of the total power input that is propagated as a surface wave is a maximum. The total power in the surface wave will be evaluated first. Due to symmetry, the total power carried by the surface wave is four times the power carried by the surface wave in the region $x>0$; $z<0$. From (21), the total power in the surface wave per unit width of the screen is obtained as

$$P_s = 4\int_{-\infty}^{0} \hat{x}\cdot[\boldsymbol{E}^s(x,z)\times \boldsymbol{H}^{s*}(x,z)]dz$$
$$= \frac{k\sin\alpha}{4}e^{-2kd\tan\alpha}. \tag{32}$$

The total power radiated in the region $-\infty<z<0$, is easily computed by asymptotically evaluating (23). For this purpose set

$$x = \rho\cos\theta$$
$$z = \rho\sin\theta. \tag{33}$$

Using (33), (23a) and (23b) become

$$E_y(x,z) = \frac{1}{2\pi}\int_c -\frac{k\sin^2\tau}{2[\cos^2\tau-\sec^2\alpha]} \cdot e^{ikd\sin\tau}e^{ik\rho\cos(\tau+\theta)}d\tau \tag{34a}$$

$$H_y(x,z) = \frac{1}{2\pi}\int_c -\frac{k\tan\alpha\sin\tau}{2[\cos^2\tau-\sec^2\alpha]} \cdot e^{ikd\sin\tau}e^{ik\rho\cos(\tau+\theta)}d\tau. \tag{34b}$$

For $k\rho>>1$, (34a) and (34b) are easily evaluated asymptotically with the result

$$E_y(x,z) = \frac{e^{i(k\rho-\pi/4)}}{\sqrt{2\pi k\rho}}\frac{k\sin^2\theta}{2(\sec^2\alpha-\cos^2\theta)}e^{-ikd\sin\theta} \tag{35a}$$

$$H_y(x,z) = \frac{e^{i(k\rho-\pi/4)}}{\sqrt{2\pi k\rho}}\frac{k\tan\alpha\sin\theta}{2(\sec^2\alpha-\cos^2\theta)}e^{-ikd\sin\theta}. \tag{35b}$$

Using (33) and (31), it is easily shown that for $k\rho\gg 1$

$$H_\rho = 0;\ H_\theta = E_y;\ E_\rho = 0;\ E_\theta = -H_y. \tag{36}$$

Hence, the outward power flow per unit area at angle θ is obtained from (35a) and (35b) as

$$S = \hat{\rho}\cdot\boldsymbol{E}\times\boldsymbol{H}^* = |E_y|^2+|H_y|^2$$
$$= \frac{k}{8\pi\rho}\frac{\sin^2\theta}{(\sec^2\alpha-\cos^2\theta)}. \tag{37}$$

Therefore, the total power radiated per unit width of the screen in the region $-\infty<z<0$ is obtained as

$$P_{1r} = \int_0^\pi \frac{k}{8\pi\rho}\frac{\sin^2\theta}{(\sec^2\alpha-\cos^2\theta)}\rho d\theta$$
$$= \frac{k}{16\pi}\int_0^{2\pi}\frac{\sin^2\theta}{(\sec^2\alpha-\cos^2\theta)}d\theta. \tag{38}$$

By setting $w=e^{i\theta}$ in (38) and calculating the sum of the residues of the poles of the resultant integrand within the unit circle, (38) may be evaluated with the result

$$P_{1r} = \frac{k}{8}(1-\sin\alpha). \tag{39}$$

It is evident that (39) represents the total power transmitted through the unidirectionally conducting screen. When the direction of conduction of the screen coincides with the line source, that is when $\alpha=\pi/2$, since the incident electric field is entirely parallel to the direction of the wires composing the screen, there should be complete reflection of the incident field from the line source and no transmission of power through the screen.

Also, when the line source is perpendicular to the direction of the wires of the screen, that is when $\alpha=0$, there should be complete transmission of the power from the line source incident on the screen. These facts are verified by (39), where $k/8$ is easily seen to represent the total power from the line source incident on the screen, and is equal to half the total power radiated from the isolated line source. Notice that the power radiated in the region $-\infty<z<0$ is independent of the position of the line source with respect to the screen.

By substituting (33) in (25a) and (25b) and evaluating the resulting integrals asymptotically for $k\rho\gg 1$, it is found that

$$E_y(\rho,\theta) = \frac{e^{i(k\rho-\pi/4)}}{\sqrt{2\pi k\rho}}\frac{k}{2}\left[e^{-ikd\sin\theta}+\frac{\tan^2\alpha e^{ikd\sin\theta}}{(\cos^2\theta-\sec^2\alpha)}\right] \quad (40a)$$

$$H_y(\rho,\theta) = \frac{e^{i(k\rho-\pi/4)}}{\sqrt{2\pi k\rho}}\frac{k}{2}\frac{\tan\alpha\sin\theta}{(\cos^2\theta-\sec^2\alpha)}e^{ikd\sin\theta}. \quad (40b)$$

The asymptotic evaluation of (24a) yields the same result as (40a), as might be anticipated due to the symmetry of (24a) and (25a) with respect to z and d. Hence, (40a) and (40b) are uniformly valid for $z>0$. On account of (36) and (37), the outward power flow per unit area at an angle θ is obtained from (40a) and (40b) as

$$S = |E_y|^2+|H_y|^2 = \frac{k}{8\pi\rho}\left[1+\frac{\tan^2\alpha}{(\sec^2\alpha-\cos^2\theta)} - \frac{2\tan^2\alpha\cos(2kd\sin\theta)}{(\sec^2\alpha-\cos^2\theta)}\right]. \quad (41)$$

Hence, the total power radiated per unit width of the screen in the region $z>0$ becomes

$$P_{2r} = \int_0^\pi \frac{k}{8\pi}\left[1+\frac{\tan^2\alpha}{(\sec^2\alpha-\cos^2\theta)} - \frac{2\tan^2\alpha\cos(2kd\sin\theta)}{(\sec^2\alpha-\cos^2\theta)}\right]d\theta. \quad (42)$$

If the second term is evaluated as in (38), (42) reduces to

$$P_{2r} = \frac{k}{8}(1+\sin\alpha) - \frac{k\tan^2\alpha}{4\pi}\int_0^\pi\frac{\cos(2kd\sin\theta)}{(\sec^2\alpha-\cos^2\theta)}d\theta. \quad (43)$$

Therefore, using (32), (39) and (43), the launching efficiency is obtained as

$$\eta_0 = \frac{\sin\alpha e^{-2kd\tan\alpha}}{1-\dfrac{2\sin^2\alpha I}{\pi}+\sin\alpha e^{-2kd\tan\alpha}} \quad (44)$$

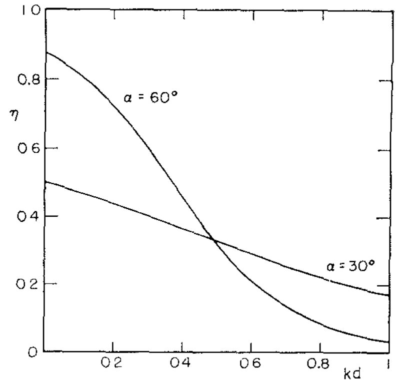

Fig. 3—Efficiency of excitation vs kd.

where

$$I = \int_0^{\pi/2}\frac{\cos(2kd\sin\theta)}{(1-\cos^2\alpha\cos^2\theta)}d\theta. \quad (45)$$

For two values of α, namely $\alpha=30°$ and $60°$, I is evaluated by numerical integration, and hence, the value of η_0 found for the range of values of kd from 0 to 1. It is seen from Fig. 3 that the highest efficiency is obtained for $kd=0$ for all values of α. The efficiency of excitation for larger values of α is larger than that for smaller values of α; it is found that the efficiency decreases with the distance of the line source from the screen, at a more rapid rate for large values of α. For $kd=0$, I becomes $\pi/2\sin\alpha$ and therefore $\eta_{0\max}=\sin\alpha$. Hence, for $kd=0$, the value of η_0 increases as α is increased.

Purity of the Surface Wave Field Near the Guiding Surface

From the expressions for the field components in (18)–(20), the components of the surface-wave field (21), were obtained by evaluating the residue at the pole $\zeta=k\sec\alpha$. It is now desirable to obtain the leading term of the asymptotic series of the radiation field in inverse powers of kx and examine whether or not it can be nullified by a proper choice of the height d of the line source from the guiding surface. In (18)–(20), the integral along the contour embracing the branch cut (Fig. 1), contributes to the radiation field and this contribution can be obtained as a series in inverse powers of x by expanding the integrand in a Taylor series and integrating term by term. The result is

1) $-\infty<z<0$

$$E_y(x,z) = \frac{ike^{i(kx-\pi/4)}}{(2kx)^{3/2}\sqrt{\pi}\tan^2\alpha} \quad (46a)$$

$$H_y(x,z) = \frac{(z-d)k^2e^{i(kx-\pi/4)}}{(2kx)^{3/2}\sqrt{\pi}\tan\alpha}. \quad (46b)$$

2) $0<z<\infty$

$$E_y(x,z) = -\frac{ike^{i(kx-\pi/4)}(1+2dzk^2\tan^2\alpha)}{\sqrt{2\pi}(kx)^{3/2}\tan^2\alpha} \quad (47a)$$

$$H_y(x,z) = -\frac{(z+d)k^2e^{i(kx-\pi/4)}}{(2kx)^{3/2}\sqrt{\pi}\tan\alpha}. \quad (47b)$$

In deriving (46) and (47), it is assumed that $\alpha>0$. An examination of (46) and (47) reveals that there is no value of d even for $z=0$, for which (46) and (47) vanish. This situation is to be contrasted with that for the problem of a magnetic line source over an impedance plane treated by Cullen,[5] who has found an optimum height of the line source which makes the leading term in the radiation field of $0[(kx)^{-3/2}]$ vanish for any z. When $z=0$ and the line source is situated at the optimum height, even the term of $0[(kx)^{-5/2}]$ vanishes so that a nearly pure surface wave field is obtained near the guiding surface. In the present problem, it is found that the radiation field near the guiding surface is $0[(kx)^{-3/2}]$ and a pure surface wave field in the sense of Cullen is not obtained.

Excitation of Surface Waves on a Semi-Infinite Unidirectionally Conducting Screen

In this section, it is assumed that the unidirectionally conducting screen extends only from $x=0$ to $x=\infty$ with the line source located at $x=a$, $z=0$. The fields due to the isolated line source are easily seen to be given by the vector potential

$$A^i(x, z) = \frac{\hat{y}}{2\pi}\int_{-\infty}^{\infty} \frac{i}{2\xi} e^{i\zeta(x-a)+i\xi|z|}d\zeta. \tag{48}$$

As before, the scattered fields are derivable from the vector potential $A^s(x, z)$ given in (13). Due to the geometry, $A^i(x, z)$, $A^s(x, z)$ are even functions of z and, hence, it is sufficient to consider only the region $z>0$. Boundary conditions (5) and (6) on the screen are seen to be automatically satisfied with the help of (48), (13) and (14). Using (15), (48), (7) and (3), it follows that

$$E_\xi(x, z) = \frac{1}{2\pi}\int\left[\frac{i\cos^2\alpha}{k}(k^2\sec^2\alpha - \zeta^2)f(\zeta) - \frac{k\sin\alpha}{2\xi}e^{-i\zeta a}\right]e^{i\zeta x+i\xi z}d\zeta. \tag{49}$$

Since boundary condition (4) gives $E_\xi(x, 0)=0$ for $x>0$, it follows from (49) that

$$\frac{i\cos^2\alpha}{k}(k^2\sec^2\alpha - \zeta^2)f(\zeta) - \frac{k\sin\alpha}{2\sqrt{k^2-\zeta^2}}e^{-i\zeta a} = u^+(\zeta) \tag{50}$$

where $u^+(\zeta)$ is a function which is regular in the upper half-plane $\text{Im}\,\zeta > -\text{Im}\,k$. Further, from (13) and (14), it is found that

$$H_\eta^s(x, z) = \frac{1}{2\pi}\int i\xi f(\zeta)e^{i\zeta x+i\xi z}d\zeta. \tag{51}$$

Since $A^s(x, z)=A^s(x, -z)$, it is seen from (14) that $H_\eta^s(x, 0)=0$ for $x<0$.

Hence, it is obvious from (51) that

$$\sqrt{k^2-\zeta^2}\,f(\zeta) = L^-(\zeta) \tag{52}$$

where $L^-(\zeta)$ is a function regular in the lower half-plane $\text{Im}\,\zeta<\text{Im}\,k$. Eliminating $f(\zeta)$ from (50) and (52) and rearranging the resulting expression, it may be derived that

$$\frac{i\cos^2\alpha}{k}(k\sec\alpha-\zeta)\frac{L^-(\zeta)}{\sqrt{k-\zeta}} - \frac{k\sin\alpha}{2(k\sec\alpha+\zeta)}\left[\frac{e^{-i\zeta a}}{\sqrt{k-\zeta}} - \frac{e^{ika\sec\alpha}}{\sqrt{k+k\sec\alpha}}\right] = \frac{u^+(\zeta)\sqrt{k+\zeta}}{(k\sec\alpha+\zeta)} + \frac{k\sin\alpha e^{ika\sec\alpha}}{2(k\sec\alpha+\zeta)\sqrt{k+k\sec\alpha}}. \tag{53}$$

It is seen that the left side of (53) is regular in the lower half-plane $\text{Im}\,\zeta<\text{Im}\,k$ and the right side is regular in the upper half-plane $\text{Im}\,\zeta>-\text{Im}\,k$. Both are regular in the strip $|\text{Im}\,\zeta|<\text{Im}\,k$. Hence, by the arguments of the Wiener-Hopf procedure, (53) is seen to define an integral function. In order to get a unique solution, the current at the end of the wires composing the screen should be required to vanish as $x^{1/2}$. It therefore follows from (51) and (52) that $L^-(\zeta)\sim\zeta^{-3/2}$ as $|\zeta|\to\infty$. Hence, from the left side of (53), the integral function is seen to vanish as $\zeta\to\infty$; therefore, its value is zero. From (53), (52) and (13), it is easily derived that

$$A^s(x, z) = \frac{1}{2\pi}\int\frac{k^2\sin\alpha\, e^{i\zeta x+i\xi z}}{2i\cos^2\alpha\sqrt{k+\zeta}\,(k^2\sec^2\alpha-\zeta^2)}\cdot\left(\frac{e^{-i\zeta a}}{\sqrt{k-\zeta}} - \frac{e^{ika\sec\alpha}}{\sqrt{k+k\sec\alpha}}\right)d\zeta. \tag{54}$$

From (10) and (54), $E_y(x, z)$ and $H_y(x, z)$ are obtained with the help of (8), (3) and (1) with the result

$$E_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-\frac{ke^{-i\zeta a}}{2\xi} - \frac{k^3\tan^2\alpha e^{ika\sec\alpha}}{2\sqrt{k+\zeta}(k^2\sec^2\alpha-\zeta^2)\sqrt{k+k\sec\alpha}}\right]\cdot e^{i\zeta x+i\xi z}d\zeta + \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{k^3\tan^2\alpha e^{i\zeta(x-a)+i\xi z}}{2\sqrt{k^2-\zeta^2}(k^2\sec^2\alpha-\zeta^2)}d\zeta \tag{55}$$

$$H_y(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} - \frac{k^2\tan\alpha}{2(k^2\sec^2\alpha-\zeta^2)}\frac{\sqrt{k-\zeta}}{\sqrt{k+k\sec\alpha}}\cdot e^{ika\sec\alpha}e^{i\zeta x+i\xi z}d\zeta + \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{k^2\tan\alpha}{2(k^2\sec^2\alpha-\zeta^2)}e^{i\zeta(x-a)+i\xi z}d\zeta. \tag{56}$$

[5] A. L. Cullen, "The excitation of plane surface waves," *Proc. IEE*, vol. 101, pt. 4, p. 225; February, 1954. (Monograph No. 93R.)

The contour for the integrals in (55) and (56) is shown in Fig. 1. The first integral in (55) and (56) is evaluated by closing the contour in the upper half-plane for $x>0$ and in the lower half-plane for $x<0$. Similarly, the second integral in (55) and (56) is evaluated by closing the contour in the upper half-plane for $x>a$ and in the lower half-plane for $x<a$. The contribution to each integral arises from a pole and a branch-cut integral and the surface-wave term, as before, is given by the residue at the poles. Taking the contribution of only the poles, the evaluation of (55) and (56) yields

$$E_y^s(x, z) = 0$$

$$H_y^s(x, z) = 0 \qquad \text{for } -\infty < x < 0 \tag{57}$$

$$E_y^s(x, z) = -\frac{k \sin \alpha}{4} e^{ika \sec \alpha - k \tan \alpha z} \cdot \left[e^{-ik \sec \alpha x} - i\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\, e^{ik \sec \alpha x}\right]$$

$$H_y^s(x, z) = -\frac{ik \sin \alpha}{4} e^{ika \sec \alpha - k \tan \alpha z} \cdot \left[e^{-ik \sec \alpha x} - i\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\, e^{ik \sec \alpha x}\right]$$

$$\text{for } 0 < x < a \tag{58}$$

$$E_y^s(x, z) = -\frac{k \sin \alpha}{4} e^{ik \sec \alpha x - k \tan \alpha z} \cdot \left[e^{-ika \sec \alpha} - i\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\, e^{ika \sec \alpha}\right]$$

$$H_y^s(x, z) = -\frac{ik \sin \alpha}{4} e^{ik \sec \alpha x - k \tan \alpha z} \cdot \left[e^{-ika \sec \alpha} - i\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\, e^{ika \sec \alpha}\right]$$

$$\text{for } a < x < \infty. \tag{59}$$

An examination of (57)–(59) reveals that a surface wave starts at $x=a$ and travels in both the positive and negative x directions with a phase velocity less than the free space velocity by a factor of $\cos \alpha$. A part of the surface wave traveling in the negative x direction gets reflected at $x=0$, the reflection coefficient being

$$-i\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

and the remaining part is converted into the radiation field. As before, it is evident from (57)–(59), (1), and (3) that the $E_\xi(x, z) = H_\xi(x, z) = 0$ for the surface wave field.

It is now desired to calculate the total power transmitted by the surface wave and that carried by the radiation field per unit width of the screen and thus calculate the efficiency of excitation. From (3) and (59) the total power carried by the surface wave per unit width of the screen is obtained as

$$P_s = \int_0^\infty \hat{x} \cdot [\bar{E}(x, z) \times H^{s*}(x, z)] dz$$

$$= \frac{k \sin \alpha}{16}\left[1 + \frac{1-\cos\alpha}{1+\cos\alpha} + 2\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \sin(2ka \sec \alpha)\right]. \tag{60}$$

It is obvious that P_s is a maximum for a given α when $2ka \sec \alpha = \pi/2$. This corresponds to the situation when the surface wave traveling in the $-x$ direction, after getting reflected at $x=0$, arrives at $x=a$ in phase with the surface wave traveling in the $+x$ direction.

To calculate the total power carried by the radiation field for unit width of the screen, it is convenient first to substitute (22) and (33) in (55) and (56) with the result

$$E_y(\rho, \theta) = \frac{1}{2\pi}\int \left[\frac{ke^{-ika\cos\tau}}{2} + \frac{k \tan^2 \alpha e^{ika \sec \alpha}\sqrt{1-\cos\tau}}{2\sqrt{1+\sec\alpha}(\sec^2\alpha - \cos^2\tau)}\right] e^{ik\rho\cos(\theta-\tau)} d\tau + \frac{1}{2\pi}\int -\frac{k}{2}\tan^2\alpha \frac{e^{-ika\cos\tau}}{(\sec^2\alpha - \cos^2\tau)} \cdot e^{ik\rho\cos(\theta-\tau)} d\tau \tag{61}$$

$$H_y(\rho, \theta) = \frac{1}{2\pi}\int \frac{k}{2} \frac{\tan\alpha \sin\tau}{\sqrt{1+\sec\alpha}} \frac{\sqrt{1-\cos\tau}}{(\sec^2\alpha - \cos^2\tau)} \cdot e^{ika\sec\alpha} e^{ik\rho\cos(\theta-\tau)} d\tau + \frac{1}{2\pi}\int -\frac{k}{2} \frac{\tan\alpha\sin\tau}{(\sec^2\alpha - \cos^2\tau)} \cdot e^{-ika\cos\tau} e^{ik\rho\cos(\theta-\tau)} d\tau. \tag{62}$$

For $k\rho \gg 1$, (61) and (62) are evaluated asymptotically to yield the following result for the radiation field:

$$E_y(\rho, \theta) = \frac{e^{i(k\rho - \pi/4)}}{\sqrt{2\pi k\rho}} \frac{k}{2}\left[\frac{\sin^2\theta}{(\sec^2\alpha - \cos^2\theta)} e^{-ika\sec\theta} + \frac{\tan^2\alpha}{\sqrt{1+\sec\alpha}} \frac{\sqrt{1-\cos\theta}}{(\sec^2\alpha - \cos^2\theta)} e^{ika\sec\alpha}\right] \tag{63}$$

$$H_y(\rho, \theta) = \frac{e^{i(k\rho - \pi/4)}}{\sqrt{2\pi k\rho}} \frac{k}{2}\left[\frac{\tan\alpha\sin\theta}{\sqrt{1+\sec\alpha}} \frac{\sqrt{1-\cos\theta}}{(\sec^2\alpha - \cos^2\theta)} \cdot e^{ika\sec\alpha} - \frac{\tan\alpha\sin\theta}{(\sec^2\alpha - \cos^2\theta)} e^{-ika\cos\theta}\right]. \tag{64}$$

Hence, the outward power flow per unit area at an angle θ is obtained from (36), (63) and (64) as

$$S = \hat{\rho}\cdot \boldsymbol{E}\times \boldsymbol{H}^* = |E_y|^2 + |H_y|^2 = \frac{k}{8\pi\rho}\left[\frac{\sin^2\theta}{(\sec^2\alpha - \cos^2\theta)} + \frac{\tan^2\alpha}{(1+\sec\alpha)}\frac{(1-\cos\theta)}{(\sec^2\alpha - \cos^2\theta)}\right]. \quad (65)$$

Therefore, the total radiated in the region $z>0$ is obtained as

$$P_r = \frac{k}{8\pi}\int_0^{\pi}\left[\frac{\sin^2\theta}{(\sec^2\alpha - \cos^2\theta)} + \frac{\tan^2\alpha}{(1+\sec\alpha)}\frac{(1-\cos\theta)}{(\sec^2\alpha - \cos^2\theta)}\right]d\theta. \quad (66)$$

By evaluating the integral in (66), in the same manner as in (58), it is found that

$$P_r = \frac{k}{8}\left[1 - \frac{\sin\alpha}{(1+\cos\alpha)}\right]. \quad (67)$$

Using (67) and (60), the launching efficiency is obtained as

$$\bar{\eta} = \frac{\frac{\sin\alpha}{2}\left[1 + \frac{1-\cos\alpha}{1+\cos\alpha} + 2\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\sin(2ka\sec\alpha)\right]}{1 - \frac{\sin\alpha}{1+\cos\alpha} + \frac{\sin\alpha}{2}\left[1 + \frac{1+\cos\alpha}{1+\cos\alpha} + 2\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\sin(2ka\sec\alpha)\right]}. \quad (68)$$

The power in the surface wave is maximized if a is chosen such that $2ka \sec\alpha = (2m-1)\pi/2$ (m is an integer) and the maximum value of the launching efficiency becomes

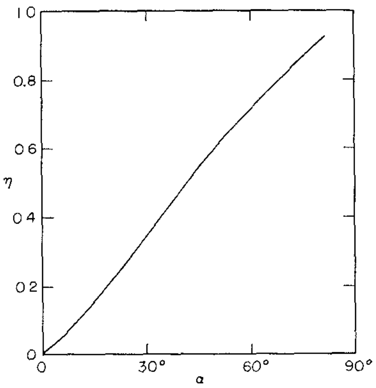

Fig. 4—Efficiency of excitation vs α.

$$\bar{\eta}_{\max} = \frac{\frac{\sin\alpha}{2}\left[1 + \frac{1-\cos\alpha}{\sin\alpha}\right]^2}{1 - \frac{\sin\alpha}{1+\cos\alpha} + \frac{\sin\alpha}{2}\left[1 + \frac{1-\cos\alpha}{\sin\alpha}\right]^2} = \frac{\sin\alpha(1+\sin\alpha)}{(1+\cos\alpha)(2-\cos\alpha)}. \quad (69)$$

The value of $\bar{\eta}_{\max}$ is plotted in Fig. 4 as a function of α. It is seen that $\bar{\eta}_{\max}$ increases nearly linearly with α. It is to be noted that it is not physically meaningful to have the line source in the plane of screen for $\alpha = \pi/2$.

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